

# Package ‘bellreg’

June 17, 2024

**Title** Count Regression Models Based on the Bell Distribution

**Version** 0.0.2.1

**Description** Bell regression models for count data with overdispersion. The implemented models account for ordinary and zero-inflated regression models under both frequentist and Bayesian approaches. Theoretical details regarding the models implemented in the package can be found in Castella et al. (2018) <[doi:10.1016/j.apm.2017.12.014](https://doi.org/10.1016/j.apm.2017.12.014)> and Lemonte et al. (2020) <[doi:10.1080/02664763.2019.1636940](https://doi.org/10.1080/02664763.2019.1636940)>.

**License** MIT + file LICENSE

**Encoding** UTF-8

**RoxygenNote** 7.3.1

**Biarch** true

**URL** <https://github.com/fndemarqui/bellreg>,  
<https://fndemarqui.github.io/bellreg/>

**BugReports** <https://github.com/fndemarqui/bellreg/issues>

**Depends** R (>= 3.4.0)

**Imports** dplyr, extraDistr, Formula, magic, MASS, methods, numbers,  
LambertW, loo, purrr, Rcpp (>= 0.12.0), Rdpack, rstan (>= 2.26.0), rstantools (>= 2.0.0)

**RdMacros** Rdpack

**LinkingTo** BH (>= 1.66.0), Rcpp (>= 0.12.0), RcppEigen (>= 0.3.3.3.0),  
rstan (>= 2.26.0), StanHeaders (>= 2.26.0)

**SystemRequirements** GNU make

**Suggests** knitr, rmarkdown, testthat

**VignetteBuilder** knitr

**NeedsCompilation** yes

**Author** Fabio Demarqui [aut, cre, cph]  
(<<https://orcid.org/0000-0001-9236-1986>>),  
Marcos Prates [ctb] (<<https://orcid.org/0000-0001-8077-4898>>),  
Fredy Caceres [ctb],  
Andrew Johnson [ctb]

**Maintainer** Fabio Demarqui <fndemarqui@est.ufmg.br>

**Repository** CRAN

**Date/Publication** 2024-06-17 19:30:02 UTC

## Contents

bellreg-package . . . . .	2
AIC.bellreg . . . . .	3
AIC.zibellreg . . . . .	4
Bell . . . . .	4
bellreg . . . . .	5
cells . . . . .	7
coef.bellreg . . . . .	7
coef.zibellreg . . . . .	8
confint.bellreg . . . . .	9
confint.zibellreg . . . . .	9
extract_log_lik . . . . .	10
faults . . . . .	11
fitted.bellreg . . . . .	11
print.summary.bellreg . . . . .	12
print.summary.zibellreg . . . . .	13
summary.bellreg . . . . .	13
summary.zibellreg . . . . .	14
vcov.bellreg . . . . .	14
vcov.zibellreg . . . . .	15
zibellreg . . . . .	15
<b>Index</b>	<b>17</b>

---

bellreg-package      *The 'bellreg' package.*

---

## Description

Bell Regression models for count data with overdispersion. The implemented models account for ordinary and zero-inflated regression models under both frequentist and Bayesian approaches. Theoretical details regarding the models implemented in the package can be found in (Castellares et al. 2018) and (Lemonte et al. 2020)

`_PACKAGE`

## References

Stan Development Team (2020). RStan: the R interface to Stan. R package version 2.19.3. <https://mc-stan.org>

Castellares F, Ferrari SL, Lemonte AJ (2018). “On the Bell distribution and its associated regression model for count data.” *Applied Mathematical Modelling*, **56**, 172 - 185. doi:10.1016/j.apm.2017.12.014.

Lemonte AJ, Moreno-Arenas G, Castellares F (2020). “Zero-inflated Bell regression models for count data.” *Journal of Applied Statistics*, **47**(2), 265-286. doi:10.1080/02664763.2019.1636940.

---

AIC.bellreg

*Akaike information criterion*

---

## Description

Akaike information criterion

## Usage

```
## S3 method for class 'bellreg'  
AIC(object, ..., k = 2)
```

## Arguments

object	an object of the class bellreg.
...	further arguments passed to or from other methods.
k	numeric, the penalty per parameter to be used; the default k = 2 is the classical AIC.

## Value

the Akaike information criterion value when a single model is passed to the function; otherwise, a data.frame with the Akaike information criterion values and the number of parameters is returned.

## Examples

```
library(bellreg)  
data(faults)  
fit1 <- bellreg(nf ~ 1, data = faults, approach = "mle")  
fit2 <- bellreg(nf ~ lroll, data = faults, approach = "mle")  
AIC(fit1, fit2)
```

---

AIC.zibellreg                      *Akaike information criterion for zibellreg objects*

---

### Description

Akaike information criterion for zibellreg objects

### Usage

```
## S3 method for class 'zibellreg'
AIC(object, ..., k = 2)
```

### Arguments

object                      an object of the class zibellreg.  
 ...                        further arguments passed to or from other methods.  
 k                            numeric, the penalty per parameter to be used; the default k = 2 is the classical AIC.

### Value

the Akaike information criterion value when a single model is passed to the function; otherwise, a data.frame with the Akaike information criterion values and the number of parameters is returned.

### Examples

```
library(bellreg)
data(cells)
fit1 <- zibellreg(cells ~ 1|1, data = cells, approach = "mle")
fit2 <- zibellreg(cells ~ 1|smoker+gender, data = cells, approach = "mle")
fit3 <- zibellreg(cells ~ smoker+gender|smoker+gender, data = cells, approach = "mle")
AIC(fit1, fit2, fit3)
```

---

Bell                                      *Probability function, distribution function, quantile function and random generation for the Bell distribution with parameter theta.*

---

### Description

Probability function, distribution function, quantile function and random generation for the Bell distribution with parameter theta.

**Usage**

```
dbell(x, theta, log = FALSE)
pbell(q, theta, lower.tail = TRUE, log.p = FALSE)
qbell(p, theta, log.p = FALSE)
rbell(n, theta)
```

**Arguments**

x                    vector of (non-negative integer) quantiles.  
theta                parameter of the Bell distribution (theta > 0).  
log, log.p          logical; if TRUE, probabilities p are given as log(p).  
q                    vector of quantiles.  
lower.tail         logical; if TRUE (default), probabilities are  $P[X \leq x]$ ; otherwise,  $P[X > x]$ .  
p                    vector of probabilities.  
n                    number of random values to return.

**Details**

Probability mass function

$$f(x) = \frac{\theta^x e^{1-e^\theta} B_x}{x!},$$

where  $B_x$  is the Bell number, and  $x = 0, 1, \dots$

**Value**

dbell gives the (log) probability function, pbell gives the (log) distribution function, qbell gives the quantile function, and rbell generates random deviates.

---

bellreg

*Bell regression model*


---

**Description**

Fits the Bell regression model to overdispersed count data.

**Usage**

```
bellreg(
  formula,
  data = NULL,
  approach = c("mle", "bayes"),
  hessian = TRUE,
  link = c("log", "sqrt", "identity"),
  hyperpars = list(mu_beta = 0, sigma_beta = 10),
  ...
)
```

**Arguments**

formula	an object of class "formula" (or one that can be coerced to that class): a symbolic description of the model to be fitted.
data	an optional data frame, list or environment (or object coercible by <code>as.data.frame</code> to a data frame) containing the variables in the model. If not found in data, the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>yrbp</code> is called.
approach	approach to be used to fit the model (mle: maximum likelihood; bayes: Bayesian approach).
hessian	hessian logical; If TRUE (default), the hessian matrix is returned when <code>approach="mle"</code> .
link	assumed link function (log, sqrt or identity); default is log.
hyperpars	a list containing the hyperparameters associated with the prior distribution of the regression coefficients; if not specified then default choice is <code>hyperpars = c(mu_beta = 0, sigma_beta = 10)</code> .
...	further arguments passed to either <code>rstan::optimizing</code> or <code>rstan::sampling</code> .

**Value**

`bellreg` returns an object of class "bellreg" containing the fitted model.

**Examples**

```
data(faults)
# ML approach:
mle <- bellreg(nf ~ lroll, data = faults, approach = "mle")
summary(mle)

# Bayesian approach:
bayes <- bellreg(nf ~ lroll, data = faults, approach = "bayes", refresh = FALSE)
summary(bayes)
```

---

cells	<i>Cells data set</i>
-------	-----------------------

---

### Description

Data set taken from (Crawley 2012) and posteriorly analyzed by (Lemonte et al. 2020). The data includes the count of infected blood cells per square millimetre on microscope slides prepared from  $n = 511$  randomly selected individuals.

### Format

A data frame with 511 rows and 5 variables:

- cells: count of infected blood cells per square millimetre on microscope slides
- smoker: smoking status of the subject (0: smoker; 1: non smoker)
- gender: subject's gender (1: male; 0: female).
- age: subject's age categorized into three levels: young ( $\leq 20$ ), mid (21 to 59), and old ( $\geq 60$ ).
- weight: body mass score categorized into three levels: normal, overweight, obese.

### References

Crawley MJ (2012). *The R Book*, 2nd edition. Wiley Publishing. ISBN 0470973927.

Lemonte AJ, Moreno-Arenas G, Castellares F (2020). "Zero-inflated Bell regression models for count data." *Journal of Applied Statistics*, **47**(2), 265-286. doi:10.1080/02664763.2019.1636940.

---

coef.bellreg	<i>Estimated regression coefficients for the bellreg model</i>
--------------	--

---

### Description

Estimated regression coefficients for the bellreg model

### Usage

```
## S3 method for class 'bellreg'  
coef(object, ...)
```

### Arguments

object	an object of the class bellreg.
...	further arguments passed to or from other methods.

### Value

a vector with the estimated regression coefficients.

**Examples**

```
data(faults)
fit <- bellreg(nf ~ lroll, data=faults)
coef(fit)
```

---

coef.zibellreg	<i>Estimated regression coefficients for zibellreg model</i>
----------------	--

---

**Description**

Estimated regression coefficients for zibellreg model

**Usage**

```
## S3 method for class 'zibellreg'
coef(object, ...)
```

**Arguments**

object	an object of the class bellreg
...	further arguments passed to or from other methods

**Value**

a list containing the the estimated regression coefficients associated with the degenerated and Bell count distributions, respectively.

**Examples**

```
data(cells)
fit <- zibellreg(cells ~ smoker + gender|smoker + gender, data = cells)
coef(fit)
```



---

confint.bellreg      *Confidence intervals for the regression coefficients*

---

**Description**

Confidence intervals for the regression coefficients

**Usage**

```
## S3 method for class 'bellreg'
confint(object, parm = NULL, level = 0.95, ...)
```

**Arguments**

object	an object of the class bellreg
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
level	the confidence level required
...	further arguments passed to or from other methods

**Value**

A matrix (or vector) with columns giving lower and upper confidence limits for each parameter. These will be labelled as  $(1-\text{level})/2$  and  $1 - (1-\text{level})/2$  in \

**Examples**

```
data(faults)
fit <- bellreg(nf ~ lroll, data = faults)
confint(fit)
```

---

confint.zibellreg      *Confidence intervals for the regression coefficients*

---

**Description**

Confidence intervals for the regression coefficients

**Usage**

```
## S3 method for class 'zibellreg'
confint(object, parm = NULL, level = 0.95, ...)
```

**Arguments**

object	an object of the class zibellreg
parm	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
level	the confidence level required
...	further arguments passed to or from other methods

**Value**

100(1-alpha)% confidence intervals for the regression coefficients

**Examples**

```
data(cells)
fit <- zibellreg(cells ~ smoker+gender|smoker+gender, data = cells, approach = "mle")
confint(fit)
```

---

extract_log_lik	<i>Extract pointwise log-likelihood from a Stan model for a bellreg model</i>
-----------------	---

---

**Description**

This function extracts the pointwise log-likelihood for a bellreg model.

**Usage**

```
extract_log_lik(object, ...)
```

**Arguments**

object	an object of the class bellreg.
...	further arguments passed to or from other methods.

**Value**

a matrix with the pointwise extracted log-likelihood associated with a bellreg model.

## Examples

```
data(faults)
fit <- bellreg(nf ~ lroll, data = faults, approach = "bayes")
loglik <- extract_log_lik(fit)

data(cells)
fit <- zibellreg(cells ~ 1|smoker+gender, data = cells, approach = "bayes", chains = 1, iter = 100)
loglik <- extract_log_lik(fit)
```

---

faults

*Faults data set*

---

## Description

Data set taken from ( ) and posteriorly analyzed by (Castellares et al. 2018). The data contains the number of faults in rolls of fabric of different lengths.

## Format

A data frame with 32 rows and 2 variables:

- nf: number of faults in rolls of fabric of different lengths.
- lroll: length of the roll.

## References

Castellares F, Ferrari SL, Lemonte AJ (2018). “On the Bell distribution and its associated regression model for count data.” *Applied Mathematical Modelling*, **56**, 172 - 185. [doi:10.1016/j.apm.2017.12.014](https://doi.org/10.1016/j.apm.2017.12.014).

Hind J (ed.) (1982). *Compound Poisson Regression Models*, volume 14 of *Lecture Notes in Statistics*. ISBN 978-0-387-90777-2, [doi:10.1007/9781461257714\\_11](https://doi.org/10.1007/9781461257714_11).

---

fitted.bellreg

*Extract Model Fitted Values*

---

## Description

This function returns the fitted values.

## Usage

```
## S3 method for class 'bellreg'
fitted(object, ...)
```

**Arguments**

object            an object of the class bellreg.  
...               further arguments passed to or from other methods.

**Value**

a vector with the fitted values (for MLE approach) or a matrix containing the posterior sample of the fitted values.

**Examples**

```
data(faults)
fit <- bellreg(nf ~ lroll, data = faults)
fitted.values(fit)
```

---

`print.summary.bellreg` *Print the summary.bellreg output*

---

**Description**

Print the summary.bellreg output

**Usage**

```
## S3 method for class 'summary.bellreg'
print(x, ...)
```

**Arguments**

x                 an object of the class summary.bellreg.  
...               further arguments passed to or from other methods.

**Value**

a summary of the fitted model.

---

```
print.summary.zibellreg
```

*Print the summary.zibellreg output*

---

### **Description**

Print the summary.zibellreg output

### **Usage**

```
## S3 method for class 'summary.zibellreg'  
print(x, ...)
```

### **Arguments**

x                    an object of the class summary.zibellreg.  
...                   further arguments passed to or from other methods.

### **Value**

a summary of the fitted model.

---

```
summary.bellreg        Summary for the bellreg model
```

---

### **Description**

Summary for the bellreg model

### **Usage**

```
## S3 method for class 'bellreg'  
summary(object, ...)
```

### **Arguments**

object                an object of the class 'bellreg'.  
...                    further arguments passed to or from other methods.

---

summary.zibellreg      *Summary for the zibellreg model*

---

**Description**

Summary for the zibellreg model

**Usage**

```
## S3 method for class 'zibellreg'  
summary(object, ...)
```

**Arguments**

object            an object of the class 'zibellreg'.  
...               further arguments passed to or from other methods.

---

vcov.bellreg            *Variance-covariance matrix for a bellreg model*

---

**Description**

This function extracts and returns the variance-covariance matrix associated with the regression coefficients when the maximum likelihood estimation approach is used in the model fitting.

**Usage**

```
## S3 method for class 'bellreg'  
vcov(object, ...)
```

**Arguments**

object            an object of the class bellreg.  
...               further arguments passed to or from other methods.

**Value**

the variance-covariance matrix associated with the regression coefficients.

**Examples**

```
data(faults)  
fit <- bellreg(nf ~ lroll, data = faults)  
vcov(fit)
```

---

vcov.zibellreg	<i>Covariance of the regression coefficients</i>
----------------	--

---

**Description**

Covariance of the regression coefficients

**Usage**

```
## S3 method for class 'zibellreg'
vcov(object, ...)
```

**Arguments**

object            an object of the class bellreg  
 ...               further arguments passed to or from other methods.

**Value**

the variance-covariance matrix associated with the regression coefficients.

**Examples**

```
data(cells)
fit <- zibellreg(cells ~ smoker + gender|smoker + gender, data = cells)
vcov(fit)
```

---

zibellreg	<i>ZiBell regression model</i>
-----------	--------------------------------

---

**Description**

Fits the Bell regression model to overdispersed count data.

**Usage**

```
zibellreg(
  formula,
  data,
  approach = c("mle", "bayes"),
  hessian = TRUE,
  link1 = c("logit", "probit", "cloglog", "cauchy"),
  link2 = c("log", "sqrt", "identity"),
  hyperpars = list(mu_psi = 0, sigma_psi = 10, mu_beta = 0, sigma_beta = 10),
  ...
)
```

**Arguments**

formula	an object of class "formula" (or one that can be coerced to that class): a symbolic description of the model to be fitted.
data	an optional data frame, list or environment (or object coercible by <code>as.data.frame</code> to a data frame) containing the variables in the model. If not found in data, the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>yppb</code> is called.
approach	approach to be used to fit the model (mle: maximum likelihood; bayes: Bayesian approach).
hessian	hessian logical; If TRUE (default), the hessian matrix is returned when <code>approach="mle"</code> .
link1	assumed link function for degenerate distribution (logit, probit, cloglog, cauchy); default is logit.
link2	assumed link function for count distribution (log, sqrt or identity); default is log.
hyperpars	a list containing the hyperparameters associated with the prior distribution of the regression coefficients; if not specified then default choice is <code>hyperpars = c(mu_psi = 0, sigma_psi = 10, mu_beta = 0, sigma_beta = 10)</code> .
...	further arguments passed to either <code>rstan::optimizing</code> or <code>rstan::sampling</code> .

**Value**

`zibellreg` returns an object of class "zibellreg" containing the fitted model.

**Examples**

```
# ML approach:
data(cells)
mle <- zibellreg(cells ~ smoker+gender|smoker+gender, data = cells, approach = "mle")
summary(mle)

# Bayesian approach:
bayes <- zibellreg(cells ~ 1|smoker+gender, data = cells, approach = "bayes", refresh = FALSE)
summary(bayes)
```



# Index

- \* **Discrete**
  - Bell, 4
- \* **Univariate**
  - Bell, 4
- \* **datasets**
  - cells, 7
  - faults, 11
- \* **distribution**
  - Bell, 4

AIC.bellreg, 3  
AIC.zibellreg, 4

Bell, 4  
BellReg (bellreg-package), 2  
bellreg, 5  
bellreg-package, 2

cells, 7  
coef.bellreg, 7  
coef.zibellreg, 8  
confint.bellreg, 9  
confint.zibellreg, 9

dbell (Bell), 4

extract\_log\_lik, 10

faults, 11  
fitted.bellreg, 11

pbell (Bell), 4  
print.summary.bellreg, 12  
print.summary.zibellreg, 13

qbell (Bell), 4

rbell (Bell), 4

summary.bellreg, 13  
summary.zibellreg, 14

vcov.bellreg, 14  
vcov.zibellreg, 15

zibellreg, 15